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INVESTIGATIONS

OF

FORMULAS

FOR THE

STRENGTH OF THE IRON PARTS

OF

STEAM MACHINERY.

BY

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INTRODUCTION.

The following investigations formed part of a supplementary course in Applied Mechanics, given by the writer, in 1866-7-8, to the Engineer Class at the U. S. Naval Academy. They are submitted to the profession with the hope that they may prove convenient and of value.

The formulæ are founded upon the principle that the different parts of a machine should be *equally* strong, and they are developed in reference to the *ultimate* strength of the material in order to leave the choice of a *factor of safety* to the judgment of the designer. The pressure, therefore, wherever it occurs, is, unless specially noticed, the *rupturing* or *crippling* pressure, and is found by multiplying the *working pressure* by the factor of safety adopted.

The factor of safety generally adopted for wrought-iron by the best authorities lies between 6 and 8. The best practice in civil engineering suggests 6; but it is evident that when the forces

act suddenly upon material the *resilience* and impressed work are greater than when they are gradually applied, and that the factor of safety, therefore, must be greater in the former than in the latter case. Again, where the material is subjected to a force both suddenly and continuously applied, it is equally evident that a still greater factor of safety should be used.

To illustrate the principle of *resilience*, let us suppose a bar of iron to be elongated by the force P, *suddenly applied*, *i.e.*, having its full intensity from the commencement of the elongation. Let the elongation corresponding to the *statical* force P be l; the *work* of the force P acting through the space l is then Pl, and is termed the *work of resilience*, and is, in general, the work which deflects, compresses, elongates, or ruptures the material, according to the circumstances of the case. Now, if the force P be *gradually* applied, little by little, beginning with an intensity *zero* and ending with an intensity P, the *mean* force will be $\frac{P}{2}$, and the elongation will still be due to P; the *work* will, therefore, be $\frac{Pl}{2}$. Hence the work of the suddenly applied force is *twice* that of the one gradually applied. The *work* of the former is, therefore, greater than is simply necessary to produce the elongation l, and will, therefore, cause an elonga-

tion greater than l, and the material will then either break or rebound and oscillate or vibrate, until, when rest ensues, the elongation will be that due to a force P, *statically* considered, unless a *set* should ensue, and then the elongation will be correspondingly greater.

The time during which a force acts has much to do with its destructive effects, but experiments have proved that there is no force, however slight, or for however short a time applied, which will not produce a certain amount of *permanent* distortion in the material. But there are limits beyond which it is unnecessary to consider these derangements of the material. In fact, within certain limits and in certain cases, the material may even be stronger after than before the derangement. The limit beyond which the material should not be strained may be thus defined: the *force applied must never be so great or so long applied as to produce a sensibly increasing set in the material.*

From such considerations as the above, the writer is led to recommend for wrought-iron that a factor of 8 be used in all cases where the forces are *suddenly* and *continuously* applied, and in all cases where the forces are *gradually* applied, that the factor be never less than 6. For steam engines the factor 8 is therefore recommended.

Professor Rankine gives the value of this factor for cast-iron, when applied to machines, as ranging from 6 to 8. From the experiments its appears that for sudden and continuous forces (such as occur in machines) 8 is the best value.

In each of the following cases an attempt has been made *first* to thoroughly examine the *practical* phase of the question, in order to obtain the *true data and conditions* of the problem, and *then* to apply the principles of mechanics and obtain the formula.

CORRIGENDA.

Page 11, line 7, for "$W =$ Weight of wheel," read "$W =$ Weight of wheel $\times$ Factor of safety."

" 13, in *Notation*, insert "$P =$ rupturing pressure by torsion on one crank-pin."

" 13, line 18, for "elacticity" read "elasticity."

" 13, $HP =$ *actual* $HP \times$ Factor of safety.

" 14, line 14, for "f" read "f_t."

" 17, line 6, for "P" read "P_1."

" 22, line 16, for "$P_1 =$ as before," read "$P_1 =$ working pressure on one crank-pin."

" 26, line 18, for "by" read "of."

" 32, line 8, after "have" add "approximately."

" 32, line 14, for "d" read "h."

" 40, line 7, for "Filed" read "Fixed."

" 40, for "center" read "centre."

" 41, line 13, for ".00000117" read ".0000 117."

WROUGHT-IRON.

Part I.

SHAFTS.

1. GENERAL FORMULÆ.

Notation.—d = Diameter of Shaft.

D = " " Cylinder.

S = Stroke of Steam Piston $= 2a$.

a = Radius of Crank.

P = Rupturing pressure at end of a, or on one crank-pin, in pounds.

p = Rupturing steam pressure in pounds per sq. inch above zero.

f = Torsional strength of material per sq. inch.

N. B. Units, pounds and inches.

$\pi = 3.1416$.

The ordinary formula for the strength of a shaft to resist rupture by torsion, supposing the section circular, is [see Rankine's App. Mech., or Moseley's Mech. Eng'ing,]:

$$Pa = \frac{\pi f d^3}{16} \qquad (1)$$

$$\therefore d = \sqrt[3]{\frac{16}{\pi} \frac{Pa}{f}} \qquad (2)$$

2. REDUCED FORMULÆ.

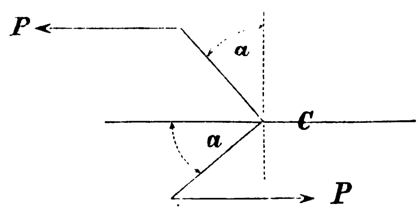
For wrought-iron, $f = 50{,}000$ lbs., as given by the best authorities here and in England; hence:

1st. *Single Lever,*

$$d = \sqrt[3]{\frac{16}{3.1416 \times 50000}} \cdot \sqrt[3]{Pa} = .0467 \sqrt[3]{Pa} \quad (3)$$

2nd. *Double Levers, or Cranks,* 90° *apart.* (Fig. 1.)

Fig. 1.



Parallel Forces.

In this case the total moment of stress is Pa (sin. a + cos. a) (a) where a is the angle made by one of the levers, when in the position of maximum *total* power, with the line of the force P, and is in this case = 45°. Hence (a) becomes

$$Pa\,(\sin. 45^\circ + \cos. 45^\circ) = Pa\sqrt{2} \qquad \text{(b)}$$

Putting (b) in place of Pa in (2) or (3), we shall have *for part most strained,*

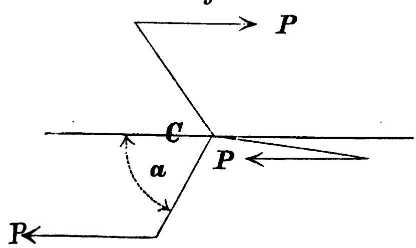
$$d = .0467 \sqrt[3]{Pa\sqrt{2}} = .0524 \sqrt[3]{Pa} \qquad (4)$$

3rd. *Triple Levers, or Cranks,* 120° *apart.* (Fig. 2.)

Parallel Forces.

The total moment in this case is (since $a = 30°$): $Pa\,(\sin. a + \sqrt{3}.\cos. a) = 2\,Pa$ (c)

Fig. 2.



Putting (c) in place of Pa in (2) or (3), we shall have, *for part most strained*,

$$d = .0467\,\sqrt[3]{2\,Pa} = .0588\,\sqrt[3]{Pa} \qquad (5)$$

4th. *Case of the Steam Engine.*

a. Screw Propellers. For the sake of simplicity, (and it involves no material error), the forces in the above cases and in the following, are supposed to remain always parallel with their first position. In this case, then, the connecting-rod is supposed to remain always parallel with its initial position, or parallel with the piston-rod. a, then becomes the angle made by one of the cranks with the piston-rod.

In this case $P = \dfrac{\pi D^2}{4}.\,p$, $a = \dfrac{s}{2}$; hence (2):

$$d = \sqrt[3]{\frac{2\,D^2\,s\,p}{f}} \qquad (6)$$

Reducing as before, we shall have for journal most strained:

$$\left.\begin{array}{lll} \text{Single Engines,} & & d = .0342 \sqrt[3]{D^2 s p} \\ \text{Double} & \text{“} & d = .0384 \sqrt[3]{D^2 s p} \\ \text{Triple} & \text{“} & d = .0431 \sqrt[3]{D^2 s p} \end{array}\right\} \quad (7)$$

b. Paddle-Wheel Engines.—These require special notice. If the steamer encounters only smooth water, as upon a river, it is evident that *both* wheels are always acting, and therefore only *one half* of the total steam pressure comes upon each of the in-board journals. If the steamer be sea-going one wheel may be acting alone, as when the vessel rolls heavily. In such case, the whole steam pressure acts upon *one* journal or wheel, and a larger shaft is required. Again, if the wheel be *overhanging*, i. e., has no *out-board* journal or bearing, the shaft must be enlarged at the out-board end to enable it to bear the weight of the wheel. This last case will be taken up in the next paragraph.

From the above considerations we obtain:

Paddle Steamers with Out-board Bearings.

River Steamers:

$$\left.\begin{aligned}\text{Single, } d &= .0342\sqrt[3]{\frac{D^2\,s\,p}{2}} = .0271\sqrt[3]{D^2\,s\,p}\\ \text{Double, } d &= .0384\sqrt[3]{\frac{D^2\,s\,p}{2}} = .0305\sqrt[3]{D^2\,s\,p}\end{aligned}\right\}\quad(8)$$

Sea-going Steamers:

$$\left.\begin{aligned}\text{Single,}\quad d &= .0342\sqrt[3]{D^2\,s\,p}\\ \text{Double,}\quad d &= .0384\sqrt[3]{D^2\,s\,p}\end{aligned}\right\}\qquad(9)$$

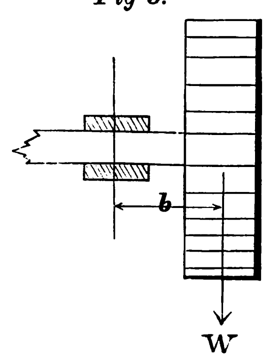
c. *Paddle-Wheel Engines. Overhanging Wheels.*

Notation.—$W.$ = Weight of wheel.

$b.$ = Distance from center of length of hub of wheel to center of outer bearing, (see fig. 3).

f' = Stress due to W per sq. inch, in pounds, upon *upper* or *lower* fibres of shaft.

Fig 3.



Now supposing only W to act, there will be only a *transverse* stress upon the material. It is well known that, within the limits of elacticity, a cylinder has twice the strength to resist torsion that it has to resist transverse stress. Hence the equation of transverse stress is (1).

$$Wb = \frac{\pi f' d^3}{32} \qquad (a)$$

$$\therefore f' = \frac{32\ W b}{\pi\ d^3} \qquad (b)$$

If P alone acts, then we shall have the stress upon the same fibres,

$$f'' = \frac{16\ Pa}{\pi\ d^3} \qquad (c)$$

If both P and W act simultaneously, the stress upon these same fibres will be the resultant of f' and f'', which act at right angles to each other. Call this resultant f:

$$\therefore f = \sqrt{f'^2 + f''^2} \qquad (d)$$

$$\therefore f^2 = \left(\frac{32\ W b}{\pi\ d^3}\right)^2 + \left(\frac{16\ Pa}{\pi\ d^3}\right)^2 \qquad (d')$$

or

$$d = \sqrt[3]{\sqrt{\frac{(32\ W b)^2 + (16\ Pa)^2}{\pi^2\ f^2}}} \qquad (10)$$

For the modulus of transverse stress we have $f_t = 36{,}000$ lbs., equal to the maximum transverse stress per sq. inch which the material will stand. For the modulus of torsional strength we have $f_s = 50{,}000$ lbs., the maximum torsional stress the fibres will stand per sq. inch. These values, f_t and f_s, are the values of f' and f'', supposing rupture to take place. Now f partakes of both

the nature of f' and f'', or of f_t and f_s; we cannot, in the absence of special experiment, do better than take a mean, therefore, of f_t and f_s as the limiting value of f.

$$\therefore f = \frac{36,000 + 50,000}{2} = 43,000 \text{ lbs.}$$

Hence,

$$d = \left(\frac{16^2}{\pi^2 f^2}\right)^{\frac{1}{6}} \sqrt[3]{\sqrt{(2Wb)^2 + (Pa)^2}} \qquad (11)$$

$$= .0491 \sqrt[3]{\sqrt{(2Wb)^2 + (Pa)^2}} \qquad (12)$$

River Steamers:

$$\left.\begin{aligned} &\text{Single, } d = .0491 \sqrt[3]{\sqrt{(2Wb)^2 + \left(\frac{Pa}{2}\right)^2}} \\ &\text{Double, } d = .0491 \sqrt[3]{\sqrt{(2Wb)^2 + \frac{1}{2}(Pa)^2}} \end{aligned}\right\} 13$$

Sea-going Steamers:

$$\left.\begin{aligned} &\text{Single, } d = .0491 \sqrt[3]{\sqrt{(2Wb)^2 + (Pa)^2}} \\ &\text{Double, } d = .0491 \sqrt[3]{\sqrt{(2Wb)^2 + 2(Pa)^2}} \end{aligned}\right\} 14$$

N. B.—In all the above cases the pressure P or p, is found by multiplying the *working pressure* by the factor of safety, and 8 is recommended as such factor. For example: if the working boiler pressure is to be 40 lbs. per guage, then we have, for a perfect vacuum, $40+15 = 55$ lbs. as the maximum

working pressure above *zero*; therefore $p=8\times 55=440$ lbs. the value to be used in the formula. We may also express the diameter in terms of the horse-power of the engine, and number of revolutions:

Let HP=horse-power of engine (*rupturing*).

n=number of revolutions (*maximum*).

$$\therefore \frac{4\,n\,Pa}{33{,}000}=12''\,HP, \therefore Pa=\frac{12\times 8250}{n}.HP$$

or

$$d=2.161\sqrt[3]{\frac{HP}{n}} \qquad (15)$$

3. Crank-pins.

Notation.—l = Length of crank-pin.

d^c = Diameter of crank-pin.

P = Total pressure required to break crank-pin.

m = Constant dependent upon mode of distribution of load.

$A = ld_c$ = *Projected area* of crank-pin necessary to *prevent heating*.

h = Pressure per sq. inch of projected area.

f_t = Transverse strength of material per sq. inch = 36,000 lbs.

f_s = Torsional strength of material per sq. inch = 50,000 lbs.

N. B.—Units, pounds and inches.

Hence,

$$l = \frac{A}{d_c} \tag{16}$$

$$A = \frac{\textit{Working pressure of one engine}}{h} = \frac{P}{h}, \tag{17}$$

But the moment of transverse strength of the crank-pin is expressed by

$$\frac{\pi f_t}{32} \cdot d_c^3$$

and the moment of applied force by

$$m\ Pl \tag{a}$$

$$\therefore \frac{\pi f_t}{32} \cdot d_c^3 = m\ Pl = m\frac{PA}{d_c} = m\frac{\pi f_s}{16a} \cdot d^3 \cdot \frac{A}{d_c} \tag{b}$$

eq. 1. $$= \frac{m\ \pi\ D^2}{4} \cdot p \cdot \frac{A}{d_c} \tag{b}$$

p and D as before.

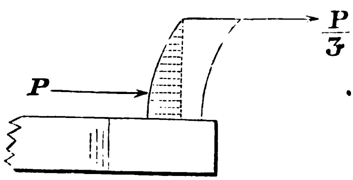
$$\therefore d_c = \sqrt[4]{\frac{2 f_s m A\ d^3}{f_t\ a}} = \sqrt[4]{\frac{8\ m\ D^2 p\ A}{f_t}} \tag{18}$$

$$= \sqrt[4]{\frac{2.78 m\ A\ d^3}{a}} = \sqrt[4]{\frac{m\ D^2 p\ A}{4{,}500}} \tag{19}$$

1st. *Single Engines—Single Crank-Pin.*

In the case of an uniform pressure along the whole length of the pin the centre of pressure is evidently at the centre of the length; but owing to the spring of brasses and pin the pressure will be almost *zero* at the free end and a maximum at the crank; and if we take it to vary *uniformly* (which seems, considering all points, the most correct law) the centre of pressure will be at one-third the length of the pin from the crank. The breaking moment will then be $\frac{1}{3}Pl$ in place of $\frac{1}{2}Pl$,—$m = \frac{1}{3}$ in place of $m = \frac{1}{2}$ (fig. 4).

Fig. 4.

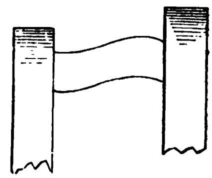


Introducing $m = \frac{1}{3}$ in 19 and there results:

$$d^c = \sqrt[4]{\frac{.93\,A\,d^3}{a}} = \sqrt[4]{\frac{D^2\,p\,A}{13{,}500}} \qquad (20)$$

If there be another crank added, so that the pin is no longer *overhanging*, it puts the pin very nearly

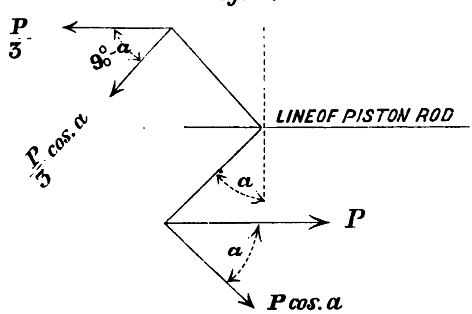
Fig. 5.



in the condition of the half of a beam *uniformily loaded* and *fixed at the two ends.* Hence in this case the breaking moment is $\frac{2}{3} \cdot \frac{1}{2} Pl = \frac{1}{3} Pl$, the same as before (fig 5).

2nd. *Double Engines. Screw Propellers.*—In this case the forward crank transmits its pressure to the after crank-pin, and hence the pressure due to this must be combined with the pressure due to its proper piston. The pressure due to its proper piston we have shown to be equivalent to $\frac{P}{3}$ at the end of the pin, and the pressure *transmitted* is equal to P acting at the same point, *i. e.*, the forward end of the pin. We have then to find the maximum resultant of $\frac{P}{3}$ and P. (Fig. 7).

Fig. 6.



Let fig. 6 represent the two cranks (90° apart), in any position indicated by the angle a; then the resultant force required is found from the following relation:

$$T^2 = \frac{P^2}{9} + P^2 \ cos.^2 \ a + \frac{2}{3} P^2 \ cos. \ a \ sin. \ a \quad (a)$$

and differentiating and placing the first diff. coeff. equal to *zero :*

$$\frac{d\,T}{d\,a} = -2 \ cos. \ a \ sin. \ a + \frac{2}{3} \ cos.^2 \ a - \frac{2}{3} \ sin.^2 a = 0$$

or,

$$3 \ sin. \ a \ cos. \ a + sin.^2 \ a = cos.^2 \ a. \quad (b)$$

$$\therefore \ cos \ a = \sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{13}}} = .9575 = cos. \, 16°46'$$

$$\therefore T = P \sqrt{\frac{1}{9} + .9575^2 + \frac{2}{3} \times .9575 \times .2885} = 1.1P. \quad (c)$$

But the pin is very nearly in the condition of the half of a beam *fixed at the ends and loaded in the centre* ; the breaking moment is therefore for *after pin :*

$$\frac{1.1Pl}{2} = .55 \ Pl.$$

or,

$$m = \frac{55}{100} \quad (d)$$

Hence (eq. 19)

after pin $\quad d_c = \sqrt[4]{\dfrac{1.53 \ A \ d^3}{a}} = \sqrt[4]{\dfrac{D^2 \, p \, A}{8182}} \quad (21)$

3d. *Triple Engine.* — Here we may take for after pin :

$$m = \frac{1.75}{2} = .875 \qquad (a)$$

$$\therefore d_c = \sqrt[4]{\frac{2.43\ A\ d^3}{a}} = \sqrt[4]{\frac{D^2 p\ A}{5143}} \qquad (22)$$

Eqs. 20, 21, and 22, require A to be determined before they can be applied; but as we shall determine l in the next paragraph it will be better to express these formulæ in terms of l.—$A = ld_c$, and this substituted in (20), (21) and (22) gives:

$$\left.\begin{array}{lll} \text{Single engines,} & d_c = \sqrt[4]{\frac{\cdot 93\ Ad^3}{a}} = \sqrt[3]{\frac{D^2 pl}{13500}} \\ \text{Double} \quad \text{"} & d_c = \sqrt[4]{\frac{1\cdot 53 Ad^3}{a}} = \sqrt[3]{\frac{D^2 pl}{8182}} \\ \text{Triple} \quad \text{"} & d_c = \sqrt[4]{\frac{2\cdot 43 Ad^3}{a}} = \sqrt[3]{\frac{D^2 pl}{5143}} \end{array}\right\} \qquad (23)$$

It may be remarked that eqs 20, 21 and 22, can be solved when h is known, since then A is also known. h is given by rules similar to those given by Rankine and Bourne, and is made to depend upon the velocity of the rubbing surfaces; but the writer has not found this method satisfactory. He submits the following method of determining the minimum length, l, which then determines (23):

4th. *Values of l.*

The *wear* and *heating* of journals must be directly proportional to the work of friction expended upon them. The hotter a journal the greater the wear, and when very hot the metal is not only torn away but *cut* away. The heat of friction in a journal is conducted away by the surrounding parts, dissipated by radiation, convected away by the lubricants, absorbed in vaporization, etc.

We have, *first*, to ascertain the proper dimensions of a journal to prevent heating and excessive wear, and, *second*, to ascertain the proper diameter to prevent rupture. Let N = number of revolutions of journal in bearings per minute:

ρ = coefficient of friction of rubbing surfaces.

P_1 = as before.

$\therefore$ work of friction $= W_f = \pi d_c N \rho P_1$, (a)

and the work per unit of projected area $(A = ld_c) =$

$$w = \frac{W_f}{ld_c} = \frac{\pi d_c N \rho P_1}{ld_c} = \frac{\pi \rho N P_1}{l} \qquad (24)$$

We see from this *that the work of friction per unit of rubbing surface is independent of the diameter of the journal.* In this discussion the projected area is taken as a measure of the surface in contact.

In (24) $\pi\rho$ being constant for the same material may be put $=$ C :

$$\therefore w = \frac{CNP_1}{l} \tag{25}$$

Or the measure of *wear and heating*, which we shall call the *coefficient of wear and heating*, may be put:

$$k = \frac{P_1 N}{l} \tag{26}$$

We can find the value of k by examining a number of cases in good practice, and thus obtain a reliable formula for l, the length of the crank-pin.

The following table gives this value of k for American and French men-of-war, as obtained from the *crank-pin journals:*

	Name of Vessel.	D.	p.	P.	N.	l.	d.	V.	h.	k
Screw Propellers.	Swatara and Class ..	36″	40	40712	80	12″	8.5″	167′	334	271142
	Saco " ..	30″	"	28274	90	9″	7.5″	157′	419	282744
	Marblehead " ..	30″	"	28274	90	9″	7″	165′	449	282744
	Wampanoig " ..	100″	"	314160	31	27″	16″	131′	727	360656
	Brooklyn " ..	61″	32	93517	60	12″	12½″	160′	625	467584
	Minnesota " ..	79½″ & 33″	28	114002	50	13″	13″	163′	675	522860
	Wabash " ..	72″	"	115041	"	16″	15″	194′	438	356256
	Merrimack " ..	72″	"	115041	"	13″	13″	159′	681	438452
	Means of 1st 4....									304847
	" 2nd 4....									446288
Paddles.	Susquehanna	70″	28	107755	15	13½″	9″	35′	887	119720
	Powhattan	70″	28	107755	15	12½″	9″	35′	958	129305
	Eutaw	58″	60	158529	24	17″	10″	63′	926	223793

French Means.

	D.	p.	P.	N.	l.	d.	V.	h.	k
4 Side lever, Side wheel	48″–76″	15–18		16–21	6″–13″	6″–12″	34′–50′	482–678	97814
7 Direct " "	44″–74″	22–24		20–30	7¾″–13″	6″–11″	46′–65′	537–818	146720
13 Screws............	44″–96″	20–56		22–135	5¾″–17¾″	7½″–17⅓″	75′–292′	425–876	401520

By examining the above table it will be seen that as an universal rule the crank-pins of screw propellers suffer from 3 to 4 times the amount of wear and heating that is expended upon those of side-wheel steamers. It is a notorious fact that the crank-pins of screw steamers are very troublesome from their liability to heat, while the reverse is the case with those of paddle steamers. The above table confirms this.

In order to make the crank-pins of screw steamers work as well as those of paddle steamers they must have a coefficient k as small as is found in those of the latter; but this would render them much larger than is practicable. We must, therefore, determine a mean value which will give *good* results. The "Eutaw," on her trial on the Potomac, when running at her maximum speed gave a coefficient $k = 224{,}000$, and still worked cool. The old frigates, with the exception of the "Wabash," always have worked hot crank-pins; the "Wabash," it will be seen, has a smaller value for k than the others of her class, her crank-pin being much *longer*. The "Swatara" and class work cool; also the "Saco" and class. The "Wampanoag" on her maximum trial is reported to have worked cool, and the speed was forced to the

utmost. It seems then that a fair value for k is 350,000.

$$\therefore l = \frac{P_1 N}{350000} \qquad (27)$$

Which gives the *minimum length* of the crank-pin. Of course it will not be necessary to use (27) except in cases where the velocity of the rubbing surfaces and the pressure are very great, as is the case with many screw propellers. We then obtain the *minimum* length of the journal allowable.

N.B.—P_1 is the *working pressure*, following full stroke upon one pin, *unbalanced;* i. e., total pressure diminished by back-pressure. In 20, 21, 22, and 23, p must be put equal to the *working pressure at commencement of stroke multiplied by the factor of safety* ($=8$) *adopted*, as in the case of shafts.

4. Cranks.

1st. *Keys.* The crank when not forged in one with the shaft is secured to it with keys, either by wrought-iron or steel, the latter being of course far preferable. It will be more convenient to determine the dimensions of the keys before determining those of the crank.

Notation. S = shearing strength of material.
= f = torsional strength of material for wrought-iron.
k = aggregate breadth of keys.
b = length of keys measured parallel with shaft, = d generally.

N.B.—Units, pounds and inches.

It is evident that the moment of shearing strength of the keys must equal the moment of torsional strength of the shaft:

$$\therefore Sk.\frac{bd}{2} = Sk.\frac{d^2}{2} = \frac{\pi f}{16}.d^3 \qquad \text{(a)}$$

Hence

$$\left.\begin{array}{ll}\text{One key} & k = \dfrac{\pi d}{8} \\[2ex] \text{Two keys} & k_1 = \dfrac{\pi d}{16}\end{array}\right\} \qquad (28)$$

Steel Keys.—We may safely put in this case:

$$\left.\begin{array}{ll}\text{One key} & k = \dfrac{\pi d}{16} \\[2ex] \text{Two keys} & k_1 = \dfrac{\pi d}{32}\end{array}\right\} \qquad (29)$$

Where k_1 = breadth of each key.

2d. *Hub. Notation.* d_x = diameter of hub, d_1 = diameter over keys.

b = length of hub along shaft.

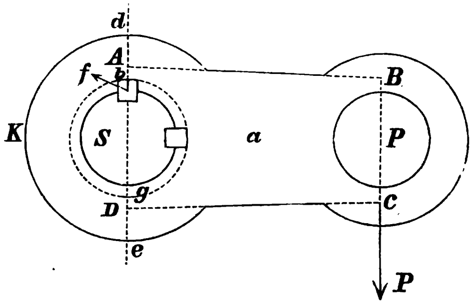
f_t = transverse strength of material.

f = torsional " "

S = shearing " "

N.B.—Units, pounds and inches.

Fig. 8.



It is evident that the hub and keys and ring of metal whose outside diameter is $d_1 = bg$, fix the crank upon the shaft in the manner of a beam fixed at one end supporting a load at the other. Hence we may consider the left hand half of the hub $d\ k\ e\ g\ b$ to act as the imbedded part of a beam whose length $= a =$ length of crank, and section $= de - bg$, strained by the force P at P. The neutral axis may be taken to pass through S, and hence the moment of resistance of the sections bd and ge should together be equal to the moment of torsional resistance of the shaft. Hence (Rankine's App. Mech., p. 317):

$$nf_t\,(bd_x^2 - bd_1^2) = \frac{\pi f}{16} . d^3 \qquad \text{(a)}$$

Where n = a constant dependent for its value upon the shape of the cross-section considered; — for a *rectangular* section $n = \frac{1}{6}$. Generally $b=d$.

$$\therefore \frac{f_t}{6}\, d\,(d_x^2 - d_1^2) = \frac{\pi f}{16} . d^3 \qquad \text{(b)}$$

or,

$$d_x = \sqrt{\frac{3\pi f d^2}{8 f_t} + d_1^2} \qquad (30)$$

For wrought-iron f_t = 36000 lbs.
f = 50000 "

And supposing there are steel keys which project half their breadth, or $\frac{k}{2}$, beyond the cylindrical surface of the shaft; then $d_1 = d\,(1 + \frac{\pi}{32})$:

$$\therefore d_x = d\sqrt{\frac{3 \times 3.1416 \times 50000}{8 \times 36000} + (1 + \frac{3.1416}{32})^2}$$

$$= 1.69d \qquad (31)$$

This is often given:

$$d_x = 1.75d \qquad (32)$$

3d. *Web. Notation.* y = breadth of web parallel with shaft at any point x.

z = depth at any point.

x = distance from centre of crank-pin to any point x.

N. B.—Units, pounds and inches.

It is evident that the moment of resistance of the material must at every point equal the moment of applied force; therefore

$$nf_t\, z^2 . y = Px \qquad \text{(a)}$$

Or,

$$y = \frac{Px}{nf_t\, z^2} \qquad (33)$$

$$\therefore\ y = \cdot 00017\, \frac{Px}{z^2} \qquad (34)$$

But $P = \dfrac{\pi f d^3}{16a}$ (eq. 1):

$$\therefore\ y = \frac{\pi f d^3 x}{16 n f_t z^2 a} = 1.64 \cdot \frac{d^3 x}{z^2 a} \qquad (35)$$

At center of shaft, $x = a$:

$$\therefore\ y = 1.64 \frac{d^3}{z^2} \qquad (36)$$

At distance, $x = \frac{a}{2}$:

$$y = \cdot 82 \frac{d^3}{z^2} \tag{37}$$

At distance, $x = z$:

$$y = 1.64 \frac{d^3}{za} \tag{38}$$

In the above equations either z or y may be assumed. It is perhaps more convenient to assume the face contour of the crank, and then z is fixed and y to be determined.

z and y must have values never less than are required to provide against shearing. The following equation determines these:

$$y = \frac{\pi f d^3}{16 S z a} = \cdot 196 \frac{d^3}{za} \tag{39}$$

4th. *Eye.* For a loose crank-pin.

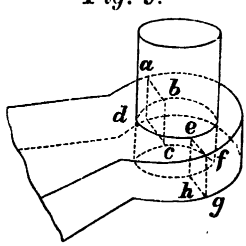
Notation.—e = diameter of eye.

d_c = diameter of crank-pin = length of eye.

N. B.—Units, pounds and inches.

If the pin is keyed into the eye, then the eye should be strong enough to equal the transverse strength of the crank-pin upon the supposition that the rupture will take place as shown by the dotted lines in Fig. 9:

Fig. 9.



$$\therefore\ nf_t d_c^2(e-d_c) = \frac{\pi}{32} f_t d_c^3 \qquad (a)$$

$$\therefore\ e-d_c = \frac{\pi f_t}{32nf_t} \cdot \frac{d_c^3}{d_c^2} = \frac{\pi}{32n} . d_c \qquad (40)$$

Or,

$$e = d_c\left(1+\frac{\pi}{32n}\right) = 1.59d_c \qquad (41)$$

5. Rods and Columns.

1st. *Pump Rods and Valve Stems.—Pillars fixed at one end.*

From Rankine's App. Mech., p. 362, we have for this case:

$$S = area = \left[\frac{P}{f}\left(1 + 4a\frac{l^2}{h^2}\right) + \frac{P}{f}\left(1 + a\frac{l^2}{h^2}\right)\right] \div 2$$

$$= \frac{P}{f}\left(1 + 2.5a\frac{l^2}{h^2}\right) \qquad (42)$$

Where

l = Length of column or rod.

d = Least diameter at *largest section.*

P = Pressure along axis which will produce rupture.

f = Constant = 36000 lbs. for wrought-iron.

a = " $= \frac{1}{3000}$ " " "

N.B.—Units, pounds and inches.

The above values were determined by Mr. Gordon from Hodgkinson's experiments.

Introducing these values in (42) and we have:

$$S = .000028P\left(1 + .00083\,\frac{l^2}{h^2}\right) \tag{43}$$

But, for a circular section,

$$S = \frac{\pi h^2}{4} = \frac{3.1416}{4}.h^2 \tag{a}$$

$$\therefore h = .00597\sqrt{P\left(1 + \cdot 00083\frac{l^2}{h^2}\right)} \tag{44}$$

For the following ratios:

$$\left.\begin{array}{lll} \frac{l}{h} = 0, & \text{Direct crushing} & h = .00597\sqrt{P} \\ \frac{l}{h} = 10 & & h = .00621\sqrt{P} \\ \frac{l}{h} = 15 & & h = .00650\sqrt{P} \\ \frac{l}{h} = 20 & & h = .00689\sqrt{P} \end{array}\right\} \tag{45}$$

The length of course is known; it is only necessary to estimate roughly the probable ratio $\frac{l}{h}$ and then use the corresponding formula of group (45); if the resulting value of h does not correspond, then choose the next formula. One trial is generally sufficient.

P is to be found by multiplying the maximum working pressure by the factor of safety adopted (8). For the stems of slide valves *P* is found by multiplying the total friction due to steam pressure and weight of valve by the factor of safety adopted (8), taking the co-efficient of friction for metal on metals wet $= \frac{3}{10}$; and it is considered best to include the *whole* area of the back of the valve undiminished by the lap over the exhaust-ports, to make up for friction of stuffing-box, etc. For example: the unbalanced pressure on the back of the valve (*i. e.*, pressure above back-pressure) is 40 lbs. per sq. inch. The area of the valve is 1200 sq. inches, weight of valve 200 lbs.

$$\therefore P = 8 \times \frac{3}{10} \times (1200 \times 40 + 200) = 115680 \text{ lbs.}$$

For pumps the value of *P* is found by multiplying the hydrostatic head-pressure and frictional resistance plus resisting pressure at discharge, by the factor of safety adopted (8).

2nd. *Piston-rods—Pillars fixed at one end.*

The above formulæ are directly applicable to to the case of a piston-rod, which is to be considered as a pillar fixed at one end only, since the

fixtures of the ends are not rigid enough to allow of its safely being considered fixed at both. A good rule for determining if a rod or pillar may be considered fixed is this: Examine the mode of fixing the rod or pillar and *judge* then whether the line of the axis before bending will, at the point of fixture, remain the same after bending; if it will then it is to be considered *fixed* at that end. But these formulæ may be more simply expressed in terms of the diameter of the cylinder and the steam pressure per sq. inch. Here then, as in the case of shafts $P = \frac{\pi}{4} \cdot D^2 p$; where D = diameter of the cylinder in inches and p = the pressure of steam (above back-pressure) in pounds per sq. inch necessary to produce rupture of the rod. This introduced in (44) gives:

$$h = .00527 D \sqrt{p\left(1 + .00083 . \frac{l^2}{h^2}\right)} \qquad (46)$$

Therefore—

$$\left.\begin{array}{lr} \text{Direct crushing} & h = .00527 D \sqrt{p} \\ \frac{l}{h} = 10 & h = .00549 D \sqrt{p} \\ \frac{l}{h} = 15 & h = .00574 D \sqrt{p} \\ \frac{l}{h} = 20 & h = .00608 D \sqrt{p} \end{array}\right\} \qquad (47)$$

p = as before 8 times the unbalanced working pressure.

3d. *Connecting Rods—Pillars free at both ends.*

In this case—

$$S = area = \frac{P}{f}\left(1 + 4a\,\frac{l^2}{h^2}\right) \tag{48}$$

as taken from Rankine's Mechanics, the values of f and a being the same as above given.

$$\therefore S = .000028P\left(1 + .00133\cdot\frac{l^2}{h^2}\right) \tag{49}$$

Circular Section. $h = .00527D\sqrt{p\left(1 + .00133.\frac{l^2}{h^2}\right)}$

(50)

Therefore—

$$\left.\begin{array}{ll} \text{Direct crushing at neck} & h = .00527D\sqrt{p} \\ \frac{l}{h} = 10 & h = .00561D\sqrt{p} \\ \frac{l}{h} = 15 & h = .00600D\sqrt{p} \\ \frac{l}{h} = 20 & h = .00654D\sqrt{p} \end{array}\right\} \tag{51}$$

Factor of safety as before = 8. P found as before.

The applications are to be made as in the last case.

6. Direct Tension. Straps and Bolts, etc.

Good wrought-iron (according to Professor Johnson's experiments, and they are confirmed by the experiments made in England), will stand a direct tensive stress of 60000 lbs. per sq. inch before rupture is produced.

Notation. Cross-sectional area of straps or bolts $= k$.

Rupturing pressure = 8 times working do. $= P$.

$$\therefore k = \frac{P}{60000} = .000017P \qquad (52)$$

For n bolts : $k = .000017 \dfrac{P}{n} = \dfrac{\pi}{4}.d_b^2$ where d_b = diameter of bolt.

$$\therefore d_b = \sqrt{\frac{.000017 \times 4}{3.1416} . \frac{P}{n}} = .00465 \sqrt{\frac{P}{n}} \qquad (53)$$

Straps for connecting-rods, forked ends, etc., must have the same *aggregate resisting sectional area* that is contained in the cross-section of the connecting-rod, forked ends, etc., *at the neck*. Let h be the diameter of such rod at the neck :

$\therefore$ aggregate cross-sectional area of straps =

$$k = \frac{\pi}{4} \cdot h^2 = .7854\, h^2 \qquad (54)$$

which must be distributed in accordance with the design.

7. Shearing. Keys, Gibs, Lugs, etc.

We have for the shearing strength of good wrought-iron $S = 50{,}000$ lbs. per sq. inch.

Hence for the aggregate resisting section of a key, gib, lug, etc.,

$$k_1 = \frac{P}{50000} = .00002P \qquad (55)$$

Care must in all cases be taken that all the reristing areas are considered; for example, in a key we have two areas, generally, to be sheared.

8. Beams.

It will be found convenient in designing machinery, particularly to the draughtsman, to have the formulæ for the strength of beams made directly applicable to each case, and we have therefore simplified these in the following cases :

1st. *General Formula.*—The following equation for the strength of a beam is general:

$$m\,Wl = nfbd^2 \qquad (56)$$

$m =$ constant dependent for its value upon the distribution of the load.

$W =$ weight necessary to produce rupture.

$l =$ length of beam.

$n =$ constant dependent for its value upon the form of cross-section.

$f =$ modulus of rupture of the material $=$ 36000 lbs.

$b =$ breadth of cross-section.

$d =$ depth " "

N. B.—Units, pounds and inches. Section uniform.

[See Rankine's Mechanics.]

2d. *Rectangular Section.* $n = \frac{1}{6}$.

Fixed at one end loaded at the other, $m = 1$:

at point of Support.

$$\therefore bd^2 = \frac{6}{36000} \cdot Wl = \cdot 00017\,Wl \qquad (57)$$

Same uniformly loaded, $m = \frac{1}{2}$:

at point of Support. $\therefore bd^2 = 000085\ Wl$ (58)

Supported at both ends, loaded at center, $m = \frac{1}{4}$:

At centre, $bd^2 = .000043\ Wl$ (59)

Same uniformly loaded, $m = \frac{1}{8}$:

At center, $bd^2 = .000021\ Wl$ (60)

Filed at both ends, loaded at center, $m = \frac{1}{8}$:

At center and points of fixture,

$$bd^2 = 000021 Wl \quad (61)$$

Same uniformly loaded,

$$m = \frac{1}{12} \text{ and } m = \frac{1}{24}:$$

At points of fixture,

$$bd^2 = .0000142\ Wl \quad (62)$$

At centre, $bd^2 = .0000071\ Wl$ (63)

3d. *Circular Section.*—In this case we may put for bd^2, d^3 = diameter of beam, and multiply the right hand member by $\frac{100}{59}$, or say $\frac{10}{6}$.

Hence:

Fixed at one end loaded at the other,

$$d^3 = .00028\ Wl \quad (64)$$

Same uniformly loaded:

$$d^3 = .00014\ Wl \qquad (65)$$

Supported at both ends, loaded at centre:

$$\text{At centre, } d^3 = .00007\ Wl \qquad (66)$$

Same uniformly loaded:

$$\text{At centre, } d^3 = .000035\ Wl \qquad (67)$$

Fixed at both ends, loaded at centre:

At points of support and centre,

$$d^3 = .000035\ Wl \qquad (68)$$

Same uniformly loaded:

At points of support,

$$d^3 = .0000233\ Wl \qquad (69)$$

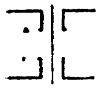
$$\text{At centre, } d^3 = .00000117\ Wl \qquad (70)$$

4th. *Elliptical Section.* In the equations of 3d put in place of d^3, a^2b, where a is the axis of the ellipse which is in the plane of flexure and b the conjugate axis,—the other quantities remaining the same.

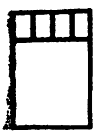
5th. *Plate-Iron Beams.* (From Fairbairn.)

Supported at both ends, loaded at centre;

Cross-section.

 Not stiffened, $W = \frac{cad}{l} = \frac{60ad}{l}$ (71)

 Stiffened, $W = \frac{75ad}{l}$ (72)

 Cellular system, $W = \frac{80\,ad}{l}$ (73)

c = constant.
a = area of section.
d = depth "
l = length between supports.
W = weight necessary to produce rupture.

N. B.—Units, inches and tons gross.

In beams for steady loads 6 may be used as the factor of safety. In other cases 8 should be used.

9. Boilers.

1st. *Strength of Plates.*

We have, from numerous experiments by Mr. Fairbairn: absolute strength of plate iron to resist tensile force = 52,486 lbs. per square inch.

Effects of Riveting.—The means of many experiments give:

Strength of plate taken at	100	= 52,486 lbs. per sq. in.
Double riveted	= 97	= 50,911 lbs. " " "
Single "	= 76	= 39,889 lbs. " " "

referring to the values of *equal* sections of the iron under the different conditions; but in a riveted joint there is, along one line of rivets, 30 per centum of the iron punched out for holes, and Mr. Fairbairn considers the value of the joint under the different conditions to be as follows:

Strength of plate taken at	100	= 52,484 lbs. per sq. in.
Double riveted	= 70	= 36,740 lbs. " " "
Single	= 56	= 29,392 lbs. " " "

From these results and other governing conditions Mr. Fairbairn gives the following as the best arrangement for riveted joints:

TABLE I.

Thickness of Plate in inches.	Diam. of Rivets in inches.		Length of Rivets from head in inches.		Dist. of Rivets from centre to centre in inches.		Quantity of lap single-riveted joints, in inches.		Quantity of lap, double-riveted joints, in inches.
$.19'' = \frac{3''}{16}$	.38		.88		1.25	6	1.25		For double-riveted joints add two-thirds of lap for single-riveted joints.
$.25'' = \frac{1''}{4}$	.50	2	1.13	4.5	1.50		1.50	6	
$.31'' = \frac{5''}{16}$	.63		1.38		1.63	5	1.88		
$.38'' = \frac{6''}{16} = \frac{3''}{8}$	.75		1.63		1.75		2.00	5	
$.50'' = \frac{1''}{2}$	.81		2.25		2.00		2.25		
$.63'' = \frac{5''}{8}$	.94	1.5	2.75	4.5	2.50	4	2.75	4.5	
$.75'' = \frac{3''}{4}$	1.13		3.25		3.00		3.25		

N. B.—The numbers after the brackets are coefficients by which, if we multiply the first column of numbers we shall obtain the figures in all the other columns respectively; *i. e.*, they merely show how many times the figures in their respective columns contain those of the first column.

2d. *Thickness of Cylindrical Boilers and Flues.*—We have the formula for strength against tensile or divellent forces:

$$e = \frac{dP}{2T} \qquad (74)$$

Where e = thickness of plates, in inches.

" d = diameter of boiler or flue, in inches.

" P = *working* pressure, in lbs. per square inch, per guage.*

" T = safe value of riveted plate, per sq. inch.

Taking a factor of safety of $\frac{1}{8}$ and referring to (1st) we get:

$T = \frac{30000}{8} = 3750$ lbs. per sq. inch for single-riveted joints.

$T = \frac{37000}{8} = 4625$ lbs. per sq. inch for double-riveted joints.

---

* In this case it is considered most convenient to introduce the working pressure instead of the rupturing pressure, since there is very little variation in practice here.

Therefore,

Single-riveted $c = \frac{dP}{7500} = .00013dP$ (74)

Double-riveted $c = \frac{dP}{9250} = .00011dP$ (75)

Against Collapse for Flues.—The following empirical formula is from Mr. Fairbairn's work:

$$P = 806{,}300 \,.\, \frac{K^{2.19}}{LD} \qquad (76)$$

When $K =$ thickness of plate, in inches.
" $L =$ length of flue, in feet.
" $D =$ diameter of flue, in inches.
" $P =$ collapsing pressure, in lbs. per sq. inch, per guage.

$$\therefore \; Log\ P = 1.5265 + 2.19\ Log\ 100K - Log\ LD \qquad (77)$$

When the necessary strength of plates in contact with the heated gases is only obtained from a greater thickness than from $\frac{5}{8}$ to $\frac{3}{4}$ of an inch, other means than the plate itself must be used to obtain it, for the iron then offers too great an impediment to the transmission of heat and suffers injury. Thus: flues of greater diameter than 30

inches and greater length than 20 feet should be stiffened by T iron hoops placed every 10 feet or so. Short flues of large diameter should likewise be braced with rods or hoops. The above remarks apply to cases where the pressures range over 60 to 100 lbs. per sq. inch.

TABLE II.—FLUES.

Formula 76—Thicknesses for Collapsing Pressure.

DIAMETER OF FLUES.	COLLAPSING PRESSURE PER SQUARE INCH.	THICKNESS OF PLATES IN INCHES.		
		For 10 ft. flue.	For 20 ft. flue.	For 30 ft. flue.
12	450 lbs. per square inch	.291	.399	.480
18		.350	.480	.578
24		.399	.548	.659
30		.442	.607	.730
36		.480	.659	.794
42		.516	.707	.851
48		.548	.752	.905

Flat Ends.—The flat ends should if possible be one-half thicker than the shell, and always be braced with rods, or gussets arranged radially and connecting the ends with the cylindrical portion.

Stays for Flat Surfaces.

The following table is taken from Mr. Fairbairn's works:

TABLE III.

No. of Experiments.	Breaking weight in tons.	Resistance per square in. in tons.	Ratio, Exp. III.—The iron stay and iron plates taken as 1,000.
III	12.5	27	1,000 to 1,000, iron and iron.
I	8.1	18.8	1,000 to 648, iron and copper, screwed.
II	10.7	23 6	1,000 to 856, iron and copper, screwed and riveted.
IV	7.2	16.1	1,000 to 576, copper and copper, screwed and riveted.

Riveting over adds 14 per cent. to the strength of a screw-stay.

The strength of flat surfaces decreases more rapidly than the area increases.

3d. *Dimensions of Stays or Bolts.*

Let P = *working* pressure in lbs. per sq. inch.

d = distance apart of stays or bolts.

T = tensile strength of wrought iron in lbs. per sq. inch = 50000.

x = diameter of stays or bolts.

N. B.—Units, pounds and inches.

And taking $\frac{1}{8}$ as the factor for safety,

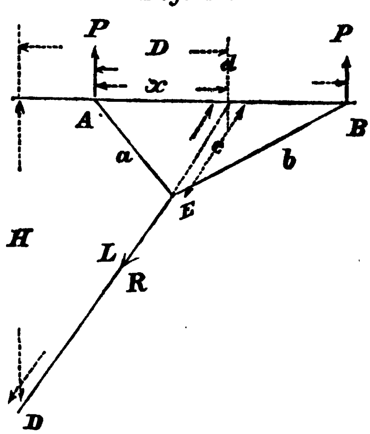
$$.7854x^2 \times \frac{50000}{8} = Pd^2$$

$$\therefore x = d\sqrt{\frac{8P}{.7854 \times 50000}} = \cdot 014 d \sqrt{P} \qquad (78)$$

Stress upon Oblique Stays.

Problem of the branch-brace. A, B, D, fig. 10, are fixed points. Suppose $A\,B$ rigid (as an angle-iron); we may then overlook the tendency to horizontal motion, and need consider only the tendency to vertical motion or motion normal to the surface $A\,B$.

Fig. 10.



Taking the origin at A, and calling the *equal* pressures at A and B, P, and the stress upon the main-brace Dd, R, equilibrium requires that :

$$2P - R\,Sin\,\widehat{Rx} = 0 \therefore R = \frac{2P}{Sin\,\widehat{Rx}} = 2P.\frac{L}{H}$$

(79)

and

$$Pd - Rx\,Sin\,\widehat{Rx} = 0 \therefore x = \frac{Pd}{R\,Sin\,\widehat{Rx}} = \frac{d}{2}$$

(80)

dE is assumed :

$$\therefore \text{ stress upon } AE = P.\frac{AE}{dE\,Sin\,\widehat{Rx}} = P.\frac{aL}{cH}$$

(81)

$$\text{" \quad " } BE = P.\frac{BE}{dE\,Sin\,\widehat{Rx}} = P.\frac{bL}{cH}$$

(82)

For the lengths of the branches :

$$a = \sqrt{c^2 + \frac{d^2}{4} - \frac{cdD}{\sqrt{D^2 + H^2}}} \qquad (83)$$

$$b = \sqrt{c^2 + \frac{d^2}{4} + \frac{cdD}{\sqrt{D^2 + H^4}}} \qquad (84)$$

4th. *Pins.*—The shearing strength of wrought-iron is about $\frac{7}{8}$th of its tensile strength; and hence observing that each pin has *two* bearings, we get

the following ratio between the diameters of the pins and braces :

$$2 \times \frac{7}{8} d^2 = d_1^2$$

$$\therefore d = d_1 \sqrt{\frac{8}{14}} = d_1 \sqrt{\frac{4}{7}} = \frac{3}{4} d_1 \qquad (85)$$

If the pin has but one bearing :

$$\frac{7}{8} d^2 = d_1^2$$

$$\therefore d = d_1 \sqrt{\frac{8}{7}} = 1.1 d_1 \qquad (86)$$

Where d = diameter of pin.
" d_1 = diameter of rod.

N.B.—In marine boilers add $\frac{1}{8}$th to $\frac{1}{4}$th of an inch to diameter of stays, pins, and bolts, to allow for corrosion.*

* This has sometimes been allowed for by reducing the value of T, but this is evidently wrong, since it assumes the depth of the corrosion to vary with the square of the diameter, whereas it is independent of it.

5th. *Strength of Metals used in Boilers.*

Mr. Fairbairn gives the following values:

Metal.	Tension, in tons per sq. inch.	Compression, in tons per sq. inch.
Wrought-iron -	23 = 52,486 lbs.	12 = 26,880 lbs.
Copper - -	16 = 35,840 "	3 = 6,720 "
Cast-iron - -	8 = 17,920 "	51 = 114,240 "

Or in round numbers:

Metal.	Tension, in lbs.	Compression, in lbs.
Wrought-iron - - -	50,000	27,000
Copper - - - -	36,000	6,700
Cast-iron - - - -	18,000	114,000

The above values for wrought-iron are taken from experiments upon plate iron. Bar iron is generally stronger than plate iron. Fairbairn gives for bar tron a tensile strength of 32 tons = 71,680 lbs. per square inch. This was probably superior iron. It may be assumed that for best quality bar iron the strength is 60,000 lbs. per square inch, as determined by Professor Johnson, of the Franklin Institute. For boiler bracing, however, when we consider the effects of welding, wear, etc., we cannot assume the iron to be more than of average quality.

For steel we have from Mr. Fairbairn :

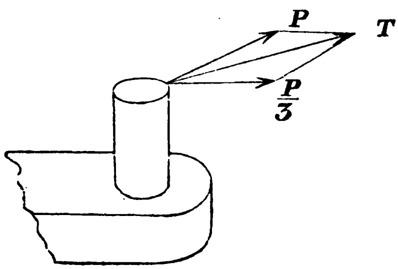
Puddled steel, - -	$T' =$	90000 lbs. per sq. inch.
Ordinary cast steel	$T' =$	128000 " " "
Sheffield steel - -	$T' =$	130000 " " "

Shearing strength :

Wrought-iron = 45,000 lbs. per sq. inch to 50,000 lbs. per sq. inch.

Cast-iron = 30,000 lbs. per sq. inch.

Fig. 7 (see p. 19).



CAST-IRON.

PART II.

10. CYLINDERS AND PIPES.

Cylinders for steam-engines and pumps should be made of the toughest cast-iron, and should be designed with a factor of safety ranging from 6 to 8, *when all forces are considered.* The following method is offered by the writer, for determining the proper thicknesses of such cylinders:

Notation.—D = diameter of cylinder.

p = *working* pressure per sq. inch, *above atmosphere.*

t = thickness of cylinder required.

f' = stress per sq. inch produced by pressure.

f'' = stress per sq. inch produced by any force tending to collapse or distort the cylinder.

f = *safe tensile strength per sq. inch.*

N. B.—Units, pounds and inches.

As suggested in the notation, the stress produced upon the material of the cylinder is made up of two forces: one resulting from the steam pressure and being a direct tensile stress; and another resulting from the distorting force, and being at the points of maximum stress similar to the stress produced in a deflected beam,—*tensive* upon the exterior fibres, and *compressive* upon the interior fibres. From the ordinary formula for the tensile stress f', we have,

$$f' = \frac{Dp}{2t} \qquad \text{(a)}$$

In order to determine f'', we shall assume, what appears almost axiomatic, that the stiffness of the cylinder is proportional to its power to resist a change of form in its cross-section from circnlar to elliptical. It is evident that with the same collapsing force the stress at the points most strained must be very nearly directly proportional to the diameter D (since, for *small deflections*, the deflecting moment is proportional to the diameter), and inversely proportional to the square of the thickness, considering the stress at the extremities of the diameter similar to that upon the fibres of a beam. Hence we may put:

$$f'' = \frac{cD}{t^2} \qquad \text{(b)}$$

where c = a constant indicating the intensity of the distorting force likely to occur.

The maximum resulting stress upon the *exterior* fibres of the cylinder must therefore be:

$$f = f' + f'' = \frac{Dp}{2t} + \frac{cD}{t^2} \qquad (87)$$

From this there results:

$$\frac{c}{f} = \frac{t^2}{D} - \frac{pt}{2f} \qquad (c)$$

$$\therefore t^2 - \frac{Dpt}{2f} = \frac{cD}{f}$$

or

$$t = \frac{Dp}{4f} + \sqrt{\frac{cD}{f} + \frac{D^2p^2}{16f^2}} \qquad (88)$$

Now good cast-iron has a tensile strength of about 18000 lbs. per sq. inch; we may therefore put $f = 2500$ lbs., giving a factor of safety between 7 and 8. c must be determined from what is considered good practice in regard to t in a particular case. In England for cylinders of 72 inches diameter, and a pressure, p, of about 30 lbs., t is made $1\frac{1}{2}$ inches. Putting these values in (c) there results:

$$\frac{c}{f} = \frac{\overline{1.5}^2}{72} - \frac{30 \times 1.5}{2 \times 2500} = .0223$$

Safe value

$$\therefore t = \cdot 0001 Dp + \sqrt{.0223 D + \frac{D^2 p^3}{100,000,000}} \qquad (89)$$

Ordinarily the last term under the radical sign is quite small compared with the first, and may be neglected; hence for such cases we have

$$\textit{Safe value } t = .0001 Dp + .15 \sqrt{D} \qquad (90)$$

This formula will give proper thicknesses in all cases; the cylinder designed by it will have the same strength and stiffness as those above quoted (72 inch English). *For pipes* subjected only to a steady tensile stress, the following is reduced from Weisbach's Mech. Engineering:

$$\textit{Safe value } t = .00016 Dp + .33 \qquad (91)$$

which provides for ordinary stiffness and a good casting.

If we neglect the quality of stiffness, i. e., make $c = o$, eq. (88) becomes

$$t = \frac{Dp}{2f} \qquad \text{(d)}$$

as in (a), as of course it should if the algebraic work is correct.

For ordinary pressures the following empirical formula is safe and agrees with good practice:

$$t = .03\sqrt{Dp} \tag{92}$$

11. Long Struts and Columns.

Ultimate Strength.

Referring to Mr. Gordon's formulæ, as before, and using the same method of reduction (Art. 5), we obtain the following:

Notation.— S = area of cross-section of strut or column.

P = rupturing force.

l = length of column.

h = least diameter of same, at largest section.

f = constant = 80,000.

a = " $= \frac{1}{400}$.

N. B.—Units, pounds and inches.

1st. *Column fixed at both ends:*

a. Any section, $$S = \frac{P}{f}\left(1 + a.\ \frac{l^2}{h^2}\right) \quad (93)$$

$$= .0000125P\left(1 + a.\ \frac{l^2}{h^2}\right) \quad (94)$$

b. Circular section, $S = \frac{\pi h^2}{4}$,

$$\therefore h = .00399\sqrt{P\left(1 + .0025\ \frac{l^2}{h^2}\right)} \quad (95)$$

c. Annular section, $S = \frac{\pi}{4}(4ht - 4t^2) = \pi(ht - t^2)$, where $t =$ thickness of metal.

$$\therefore h = .00000398\frac{P}{t}\left(1 + .0025\frac{l^2}{h^2}\right) + t \quad (96)$$

2d. *Column fixed at one end (Approximately):*

a. Any section,

$$S = .0000125P\left(1 + .00625\ \frac{l^2}{h^2}\right)$$

b. Circular section,

$$h = .00399\sqrt{P\left(1 + .00625\ \frac{l^2}{h^2}\right)}$$

c. Annular section,

$$h = .00000398\ \frac{P}{t}\left(1 + .00625\ \frac{l^2}{h^2}\right) + t$$

(97)

3d. *Column with free ends:*

$$\left.\begin{array}{l} \text{a. Any section,} \\ \quad S = .0000125\,P\left(1 + .01\dfrac{l^2}{h^2}\right) \\ \text{b. Circular section,} \\ \quad h = .00399\sqrt{P\left(1 + .01\dfrac{l^2}{h^2}\right)} \\ \text{c. Annular section,} \\ \quad h = .00000398\,\dfrac{P}{t}\left(1 + .01\dfrac{l^2}{h^2}\right) + t \end{array}\right\} \quad (98)$$

N. B.—Factor of safety should be from 6 to 8; $P = 6$ to 8 times working load or pressure. The ratio $\dfrac{l}{h}$ must be assumed in the first instance and a trial made; and if the resulting value of h does not confirm the assumed ratio another trial must be made, and so on. The formulæ (a) may be used for any symmetrical section with sufficient accuracy, by judging first in which direction the column will be likely to bend, and making h the least distance in that direction between two parallel lines passing respectively through the extreme points of compression and extension. In fixing such columns in place great care should be taken to insure the force being exactly *axial*, as (especially in the case of a solid column), a small

deviation greatly reduces the strength. In one or two cases computed by the writer, for a solid elliptical cross-section, an oblique deviation of about $\frac{1}{5}$th major diameter of the centre of pressure from the centre of symmetry caused a reduction in strength to from $\frac{1}{2}$ to $\frac{1}{3}$ the maximum strength.

12. Beams and Parts Strained as Beams.

Ultimate Strength.

The general formula is (Art. 8):

$$nfbh^2 = mWl \tag{99}$$

using the same notation as in Art. 8.

1st. *Rectangular Beams:*

From 51 experiments by Mr. Fairbairn we have $f = 36{,}000$ lbs. This is the same value used for wrought-iron (Art. 8.) This is accounted for by the fact that wrought-iron, although *tougher*, is not so *stiff* as cast-iron, and therefore cannot be taken as stronger for building purposes, in which preservation of shape and line is a requisite. Hence for beams of rectangular, circular and elliptical, cross-sections see Art. 8.

2d. *Tee Shaped Beams.*

Notation. $W =$ *breaking* weight or force at centre in *tons gross.*

$a =$ area of cross-section of *extended* flanch, in inches, at point most strained,—at centre.

$d =$ whole depth in inches, at same point, of said cross-section.

$l =$ distance between supports, in inches.

The following formulæ have been reduced from experiments by Mr. Hodgkinson and Mr. Fairbairn :

$\perp$ *Hodgkinson Beam. Extended flanch, six times compressed flanch.*

Fixed at one end, load at the other :

$$W = 6.5.\frac{ad}{l} \qquad (100)$$

Supported at both ends, load at centre :

$$W = 26.\frac{ad}{l} \qquad (101)$$

$\perp$ *Fairbairn Beam, flanch extended.*

Fixed at one end, load at the other:

$$W = 4.9.\frac{ad}{l} \tag{102}$$

Supported at both ends, load at centre:

$$W = 19.6.\frac{ad}{l} \tag{103}$$

$\top$ *Fairbairn Beam, flanch compressed, rib extended.*

From four very careful experiments by Mr. Hodgkinson (quoted in Mahan's "Civil Engineering") it appears that this beam is from 2.4 to 3.08 times stronger when the flanch is extended than when the flanch is compressed, or as an average 2.74 times stronger in the first case than in the second. Hence:

Fixed at one end, load at the other:

$$W = 1.79.\frac{ad}{l} \tag{104}$$

Supported at both ends, load at centre:

$$W = 7.15.\frac{ad}{l} \tag{105}$$

which will answer for ordinary cases. With very deep ribs this form may be still weaker.

$\perp$ *Equal Flanches.*

Fixed at one end, load at the other:

$$W = 4.\frac{ad}{l} \tag{106}$$

Supported at both ends, load at centre:

$$W = 16.\frac{ad}{l} \tag{107}$$

3d. *General Formula.* The most general form of the equation for the strength of a beam is:

$$M = \frac{fI}{y} \tag{108}$$

Where M = resultant moment of impressed forces at section considered.

" I = moment of inertia of section considered.

" f = stress per sq. inch on fibres most strained.

" y = distance from neutral axis to fibres most strained.

If we make $f =$ the *ultimate* strength as found by experiment, eq. 108 gives the ultimate strength of the beam. *This* value of f is neither that of the ultimate *compression* or ultimate *tensile* strength, as is proved by experiment. In cast-iron the difference between these two last is very great,—the strength to resist rupture by compression being about six times that to resist rupture by tension; and the value of f for *transverse* strength being between the first and last. That form of cross-section gives the strongest beam which permits the *actual* stresses of compression and tension per sq. inch on the fibres of the material to bear respectively the same ratio to the *ultimate* strengths to resist compression and to resist tension. This is nearly attained in the Hodgkinson Beam, by making the *extended* flanch *six* times greater in cross-sectional area than the compressed flanch. f varies with every cross-section. For solid sections we have seen it is equal to about 36,000 lbs. For *open work* the following is given by Mr. Barlow:

$$f = 18{,}750 + 23{,}000 \frac{H}{h} \qquad (108)$$

(Where H is the *depth of solid metal* in the section of the beam, h the *extreme depth*.)

N. B.—The weight or force W in the formulæ should be from 6 to 8 times the working load.

In all cases the cross-section at the points of support must be sufficient to resist *shearing across*.

NOTE.

The following expression for the quantity $\frac{I}{y}$ in the case of Tee Beams has been deduced by the writer:

$$\frac{I}{y}=\frac{rKd}{m_o}\times$$

$$\frac{m_o[1+mn^2+m_1n_1^2+3\{(1+n_1)^2+(n+n_1)^2m\}]-3[\{(n+n_1)m-(1+n_1)\}^2]}{(6n_1+12n)m_1+12n_1+12n+6+6mn}$$

Also,

$$y=\frac{(n_1+2n)m_1+2n_1+2n+1+nm}{2}\cdot\frac{rd}{m_o}$$

Where $m=\frac{A_2}{A_1}$, $m_1=\frac{A_3}{A_1}$, $n=\frac{d_2}{d_1}$, $n_1=\frac{d_3}{d_1}$, $r=\frac{d_1}{d}$,

$$m_o=\frac{K}{A_1}=\frac{A_1+A_2+A_3.}{A_1}$$

A_3, A_2, A_1, are respectively the areas of the rib, the extended and the compressed flanches; $d_3 =$ depth of rib, $d_2 =$ thickness of A_2, $d_1 =$ thickness of A_1, $d = d_3 + d_2 + d_1$.

For ordinary cases, $\frac{I}{y} = \frac{1}{3} Kd$ nearly, for beams of strongest form (Hodgkinson); and $f = 21000$ lbs (from Barlow's formula, making $\frac{H}{h} = \frac{1}{10}$.), $K =$ 2.25 A_2, about,

$$\therefore M = 7000\ Kd = 15750\ A_2\, d \qquad \text{(a)}$$

This does not differ much from .(100) *supra*, nor from that given by Mr. Rankine, $M =$ 16500 $A_2 d$.

13. *Shearing:*

$K =$ Aggregate area required for resistance.
$P =$ rupturing pressure.
$f_s =$ Shearing strength of material per sq. in.

N. B.—Units, pounds, and inches.

$$\therefore K = \frac{P}{f_s} = \frac{P}{30000} = .0000\ 333\ P \qquad (109)$$

14. *Tension:*

$f =$ tensile strength of material per sq. in.

$$\therefore K = \frac{P}{f} = \frac{P}{18000} = .0000\ 555\ P \quad (110)$$

ADDENDA.

Stress due to change of temperature.

1st. *General Formula.*

Notation:

$E =$ modulus of elasticity of material in pounds per sq. in.

$e =$ contraction or elongation of material, per unit of length, per degree Fahr. of change in temperature.

$t_1 =$ initial temperature, Fahr.

$t_2 =$ terminal temperature, Fahr.

$S =$ stress upon material in pounds.

$$\therefore e(t_1 - t_2) : 1 : : S : E$$

$$\therefore S = eE(t_1 - t_2) \qquad \text{(a)}$$

2nd. *Reduced Formula.*

Wrought Iron :

$$E = 28,000,000, \quad e = .00000\ 67802.$$

$$\therefore S = 190\ (t_1 - t_2) \qquad \text{(b)}$$

Cast Iron :

$$E = 17,000,000, \quad e = .00000\ 61723.$$

$$\therefore S = 105\ (t_1 - t_2) \qquad \text{(c)}$$

In cases where the parts of a structure cannot accommodate themselves freely to the contraction or expansion due to a change of temperature, the modulus of strength of the material used in designing them must be diminished by s. Thus, in ( ), if s is a stress due to *contraction*, and the part cannot accommodate itself freely to the change:

$$K = \frac{P}{f - s} \qquad \text{(d)}$$